Microeconomic Analysis - Problem Set #5

Ryan Safner

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1. Describe, in your own words, the constrained optimization problem that producers are attempting to solve. What happens at the optimum, and what does the equality of the slopes of the isoquant and isocost line *mean* economically (in English, not math)?

2. A jeweler can potentially use two inputs in her handcrafted jewelry: copper or bronze. She finds that when she minimizes her costs (i.e. she is optimizing), she uses either copper or bronze, but never both. If these are her two inputs, what must her isoquants look like?

3. Suppose a firm can hire labor at $5/hour and rent capital for $20 per hour.

   (a) Write an equation for the total cost of the firm as a function of the amount of how many workers they hire and how much capital they rent.

   (b) If the firm wants to maintain its total cost at exactly $100, find the equation of the isocost line. Graph the isocost line by solving for K.

   (c) If the firm is completely automated (i.e. using only capital), how many units of capital can they employ?

   (d) If the firm uses only labor, how many units of labor can they employ?

   (e) What is the slope of the isocost line? What does it represent?

   (f) Suppose a tax on capital makes renting capital raises the price of capital to $25 per hour. What is the new equation of the $100 isocost line, in terms of K? Graph the new isocost line on the same graph.

4. For each of the following production functions, identify whether the production process exhibits constant returns to scale, increasing returns to scale, or decreasing returns to scale:

(a) $q = 2L + 4K$

(b) $q = 6L^{0.25}K^{0.75}$

(c) $q = 2L^{0.8}K^{0.4}$

(d) $q = 2L^{0.25}K^{0.25}$

5. Mad Max's Road Warriors fix potholes in interstate highways. Max's road crews fill potholes using workers and shovels, always in 1 to 1 combinations. Using 1 worker with 1 shovel can fill 10 potholes in a day. A worker with 2 shovels can still only fill 10 potholes, as can 2 workers with 1 shovel.

  (a) What is the relationship between workers and shovels?

  (b) Write a production function for Max Max.

  (c) Draw the production isoquant corresponding to filling 20 potholes.

  (d) Add an isoquant corresponding to filling 10 potholes and one for 20 potholes.

  (e) Mad Max receives a state contract to fill in 30 potholes. If the price of shovels is $10, and wages are $5, what is the lowest cost Mad Max can fulfill the contract at? Plot this isocost line on the graph.

  (f) Add two more isocost lines to the graph, one for a total cost of $15 and one for $30. Hint: the slope doesn't change.

  (g) If the cost of renting a shovel rises from $10 to $20, what will happen to the rate at which Max Max combines workers and shovels to fill the potholes? Why?

6. Your firm builds aircraft engines using both labor (L) and power tools (K). The production function for aircraft engines is $q = 10L^{0.5}K^{0.5}$

   (a) Can you completely mechanize the production of aircraft engines?

   (b) Suppose you currently have 100 power tools. In the short run, you cannot buy or sell power tools. What is your short run production function?

   (c) Find the total product of labor, average product of labor, and marginal product of labor for using 0, 1, 2, 3, 4, and 5 workers. Round to the nearest engine. Does labor experience diminishing returns?

   (d) Sketch two graphs (roughly), one of total product, and one of marginal and average product, each with labor on the horizontal axis.

   (e) In the long run, does the production function exhibit constant returns to scale, increasing returns to scale, or decreasing returns to scale?

7. Dunder Mifflin paper company produces reams of paper each week according to the production function:

$$q = 10\sqrt{LK}$$

where:

$$MP_L = 5L^{-0.5}K^{0.5}$$
$$MP_K = 5L^{0.5}K^{-0.5}$$

They know they need to ship 1,000 reams of paper this week, and using capital costs $20, whereas labor costs $10.

(a) What is the cost-minimizing combination of labor and capital that will yield 1,000 reams of paper? Round each to the nearest whole number.

(b) What is the cost of using this combination of inputs?

(c) Now suppose that they need to double their output this week, and need to produce 2,000 reams of paper. How does their optimal combination of inputs change? (Hint: neither MRTS nor prices are changing).

(d) What is the total cost of this new level of output?